Eratosthenes' teachings with a globe in a school yard

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Abstract
A globe, in a school or university yard, which simulates the Earth's orientation in space, could be a very useful and helpful device for teaching physics, geometry, astronomy and the history of science. It would be very useful for science education to utilize the forthcoming International Year of the Planet Earth 2008 and the International Year of Astronomy 2009 by installing globes in many school and university courtyards.

Introduction

The idea that pupils in schools perform Eratosthenes' measurement of the radius of the Earth as a part of earth science [1] and physics courses [2, 3] has spread very quickly all over the world. The number of schools and universities taking part in Eratosthenes' international projects has been increasing [4] since the first invitation sent to schools by science teacher James Meinke [1, 4a]. He invited schools to perform the measurement of a shadow of a vertical stick at noon during the equinox on 21 March 1995 and to contribute their results to a shared repository [1]. With this activity the classical physics laboratory started to expand to the school yard and the wider human environment. This is fortunate, because a wider space than a classroom is necessary [5] in order to be able to repeat many other basic experiments, performed by great founders of physics in the process of the increase of human knowledge.

Enlarging the physics laboratory requires the use of specific devices to demonstrate physical phenomena and perform observations and experiments. In this article we are going to elaborate that a paradigmatic example of such an object is a globe which simulates Earth's orientation. It was devised by Rishpon and first mounted in the Clore Garden of Science at Rehovot in Israel [6]. It would be very useful for science education to utilize the forthcoming International Year of the Planet Earth 2008 [7] and the International Year of Astronomy 2009 [8] by installing Rishpon's globes in many school and university courtyards.

Strozzi's image: 'Eratosthenes teaching in Alexandria'

An interesting representation of our argument is the painting made by Bernardo Strozzi, painted around 1635, '*Erathostenes teaching in Alexandria*' (figure 1). It hangs in the Museum of Fine Arts in Montreal [9].

Looking at Strozzi's picture, one can imagine that Eratosthenes (∼276–194 BC), director of the famous Library of Alexandria, shows to a young man the page of a book where he reads that in Syene (figure 2), at the time of the summer solstice and at noon local solar time, the Sun illuminates the bottom of the well (figure 3). Syene (now

Figure 1. The picture 'Eratosthenes teaching in Alexandria' created by Bernardo Strozzi around 1635. Numerical reproduction by courtesy of the Montreal Museum of Fine Arts; No. 1959.1225.

Figure 2. The towns Alexandria and Syene are on the banks of river Nile which flows approximately along the meridian.

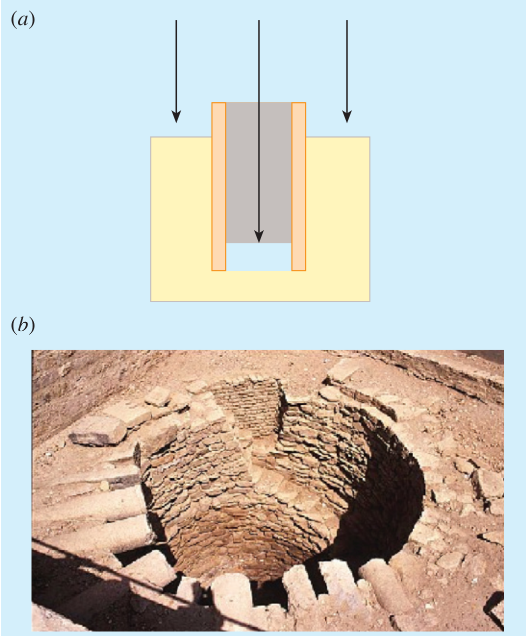

Figure 3. (a) The Sun can be seen in the bottom of the well in Syene during the Summer solstice at noon local solar time [4(e)]. (b) The well of Eratosthenes recorded by the team of students who participated in the project 'Following the footsteps of Eratosthenes' [4(b)] and traveled in 2002 from Alexandria to Aswan. The team was supported by the French Centre for Culture and Cooperation in Cairo.

Aswan) is in South Egypt, approximately at the latitude of the Tropic of Cancer.

One can imagine also that, to his student, who owns the globe, he explains that he is waiting for the next summer solstice to measure shadows of high objects in Alexandria (figure 4). From this observation, namely that, at noon during the solstice, shadows exist in Alexandria while there is no shadow in Syene at the same time, Eratosthenes derived the convincing proof that Earth is a spherical object.

In addition, Eratosthenes invented the method to measure the circumference of the Earth by measuring two quantities: (i) the incidence angle, α, of the Sun's rays (figures 4 and 5) and (ii) the distance between Alexandria and Syene, d (figure 5).

By measuring the height H of a column and the length L of its shadow, Eratosthenes found that, during the summer solstice at local solar noon in Alexandria, $\alpha = 7.2°$. He sent a caravan of camels to Syene in order to determine the distance d, by measuring the time of their travel between the two towns. The value of d

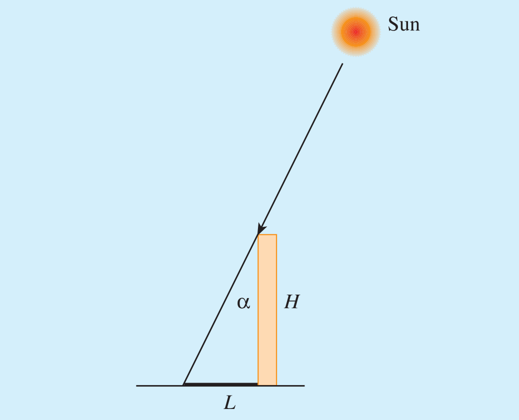

Figure 4. The incidence angle α of the Sun's rays may be determined by measuring the height of an object and the length of its shadow.

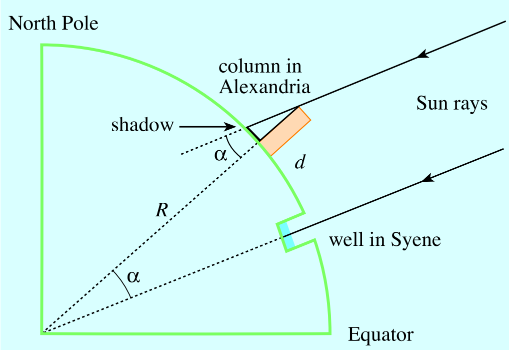

Figure 5. Using such a drawing Eratosthenes derived relation (1) for the Earth's circumference.

estimated by Eratosthenes was 5000 stadia [11]. Eratosthenes then reasoned that the ratio of d to the circumference of Earth O was equal to the ratio of the angle $\alpha = 7.2°$ to the full angle, which is 360°:

$$\frac{d}{O} = \frac{\alpha}{360°} \Rightarrow O = \frac{360°}{7.2°}d. \quad (1)$$

By substituting the value of d, one obtains

$$O = \frac{360°}{7.2°}5000 \text{ stadia} = 250\,000 \text{ stadia}. \quad (2)$$

Today, we do not know exactly what distance a stadium meant to Eratosthenes. Based on actual sizes of Greek stadiums, it must have been about 1/6 km [11]. Thus $d = (5000/6)$ km $= 833.3$ km. Consequently, $O = 41\,666$ km. This value is very close to the modern value of just over 40 000 km.

Using Archimedes' relation

$$O = 2\pi R, \quad (3)$$

where π is Archimedes' constant, one determines the value of Earth's radius $R = 6631$ km, which corresponds to Eratosthenes' value of the circumference. This result is very close to the value $R = 6370$ km of the mean radius of the Earth, which was determined later and used today.

The Archimedes relation leads us to the notion of a radian as a unit of the arc measure of an angle:

$$\text{rad} = \frac{2\pi}{360°}1° = 0.017\,45°. \quad (4)$$

By assuming that the angle α is expressed in radians,

$$\alpha = 7.2° = \pi \cdot 7.2°/180° = 0.125\,664 \text{ rad}, \quad (5)$$

Eratosthenes' relation (1) is written in the form

$$\alpha = \frac{d}{R} \Rightarrow R = \frac{d}{\alpha}, \quad (6)$$

from where the value of R directly follows. We note also from figure 4 that for small angles (measured in radians) the following relation is approximately valid:

$$\alpha = \frac{L}{H}. \quad (7)$$

But Eratosthenes' teaching spread very slowly because of the religious opposition to his thinking. It was not until the XVth and XVIth centuries that his teaching, or that of Aristarchos of Samos (~310–230 BC), who was the first to claim that the Earth spun on its axis and rotated around the Sun, found followers in the shape of Copernicus, Brahe, Kepler and Galileo.

Bernardo Strozzi painted the image of Eratosthenes in Alexandria just after Galileo Galilei was called to Rome in 1633, to face the Inquisition for the second time. Galileo was found guilty of heresy for his '*Dialogue Concerning the Two Chief World Systems*' (*Dialogo sopra i due massimi sistemi del mondo*), in which he supported the Copernican theory of the Solar system by bringing forth new observational evidence. Galileo was sent to his home near Florence where he was to be under house arrest for the remainder of his life. It is very probable that

Figure 6. The Globe in the Clore Garden of Science in Rehovot which simulates the Earth's orientation, imaged during summer (top), during equinoxes (middle) and during winter (bottom). Courtesy of Dr Moshe Rishpon.

Bernardo Strozzi, in drawing this image, used the authority of Eratosthenes in order to oppose [9] the Inquisition of the Catholic Church and to support Galilei.

A globe having Earth's orientation as an element of the learning environment

A globe, as a teaching tool in open space, might be of enormous didactic value, as demonstrated by Rishpon in the Clore Garden of Science at Rehovot in Israel [6].

The globe is positioned in such a way to simulate the Earth's orientation in space (figure 6). This implies that Rehovot is represented on the top of the terrestrial globe. The south–north axis of the globe points towards the celestial north pole. The angle between the globe axis and the local horizon is equal to the geographical latitude of Rehovot. The local horizon of the Rehovot observer and the tangent plane at the point representing Rehovot on the globe are parallel.

On the globe orientated as described above, students could observe the position of the day–night line (circle) on the Earth, at the moment of observation. Then, they could follow how it moves during the day and how fast it moves. Nowadays many airplane companies show on TV monitors in the airplane the planar presentation of the instantaneous position of the day–night line, usually taken from the US Naval Observatory [10]. The comparison of the planar and spherical presentations of the day–night circle may be a valuable introduction in teaching the mapping of a sphere to a plane.

A globe in an open space is an excellent tool to learn and remember the cause of the change of seasons. Three-dimensional globes have many advantages over usual planar textbook pictures of spheres in various positions on the Earth's orbit. For this reason a globe in a school garden is particularly useful, because students may observe various changes throughout the year.

Three characteristic orientations of the plane determined by the day–night circle with respect to the Earth's (globe's) axis are shown in figure 6. During equinoxes the Earth's axis lies in the plane determined by the day–night circle. During other days of the year the axis pierces this plane. The angle between this plane and the Earth's (globe's) axis oscillates during the year, between $23^\circ\ 36'$ and $-23^\circ\ 36'$. At noon during the solstice, if one puts small vertical columns in Syene and Alexandria on the globe, one makes a three-dimensional model of what Eratosthenes imagined in his head and drew in the plane (figure 5).

A globe having Earth's orientation as a sundial

Spherical sundial

Spherical sundials, such as the one erected in 2006 in the vicinity of the Max Valier Observatory in Italy [12], are essentially globes which simulate the Earth's orientation in space. On this particular globe there are 24 metal pins along the equator at each 15° longitude, and a pin at the intersection point of the local meridian with the equator. The shadows of the pins show whether the sun is north or south of the equator. At the equinoxes, the shadows of all pins fall along the equator. With the help of the shadows of the pins (figure 7) one can approximately determine the local solar time.

Figure 7. Hands on spherical sundial.

Time is read from an equiangular scale around the equator. Noon (12 h) is indicated at the point of intersection of the equator and the meridian of the dial's location. The shadow of a pin at this location is shortest at noon. The current solar time at the place of the sundial is read by looking for the pin with shortest shadow and its angular separation from the sundial's meridian. A spherical sundial with pins is an improved version of Jefferson's spherical sundial, which uses a movable vane to read the time [13].

Equatorial and horizontal sundial

The shadow of the globe axis on the horizontal plane may also be used to determine approximately the true local time—the solar time. It is just necessary to draw, on the horizontal surface below the globe, the timescale appropriate for the local place. This timescale is determined

Figure 8. Photograph of the equatorial sundial in the Forbidden City in Beijing, taken by Noam Sienna in 2003. Released under terms of GNU Free Documentation Licence. http://en.wikipedia.org/wiki/Sundial.

in the same way as for a traditional horizontal sundial.

To understand how one determines the timescale of a horizontal sundial one starts with the equatorial sundial. It consists of a disc and of an axis, which is normal to the plane of the disc (figure 8). The axis is parallel to Earth's axis so that the disc is parallel to the equatorial plane of the Earth, as well as to the tangential plane at the poles. It follows from this that the equatorial sundial is a simplified form of a globe which simulates Earth's orientation and has an extended axis.

The shadow of the axis on the disc is parallel to the shadow which a vertical column on Earth's pole would throw on the tangential surface around the pole. Both shadows have constant length during the day and rotate uniformly. This is because: (1) the Earth rotates uniformly, and (2) the angle between Earth's axis and Sun's rays is approximately constant during a day. During one hour the shadow of the axes rotates by an angle of 15°. This is why the hour lines on an equatorial sundial are spaced at 15° intervals. The noon line connects the disc centre and its bottom, 6 am is on the western edge, and 6 pm on the eastern edge. Angles are measured from the noon line towards the east and towards the west.

However, the length of the shadow of the disc axis on the horizontal surface below the disc changes during a day (figure 9). The

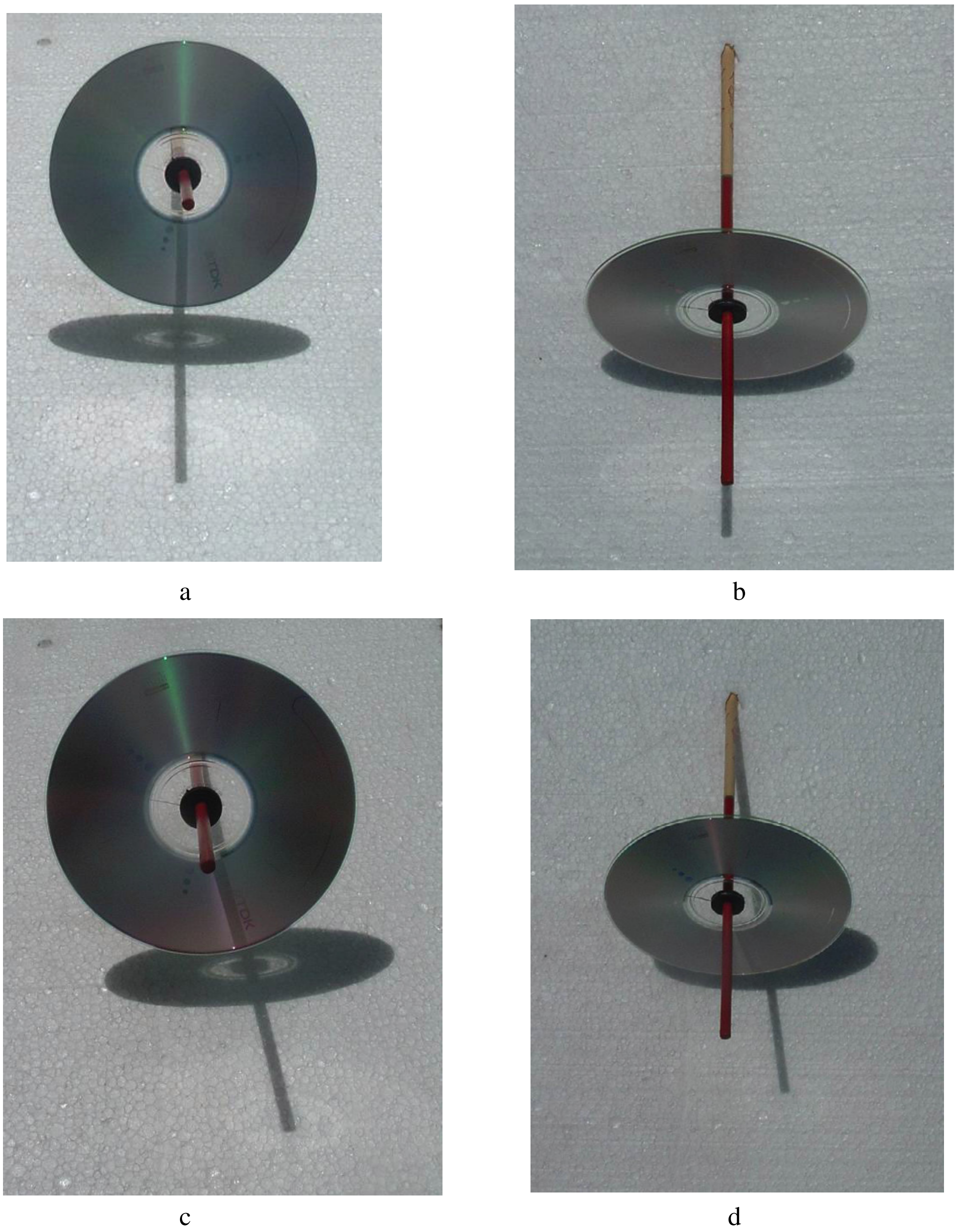

Figure 9. The images show the shadows of the disc axis on the disc and on the ground at noon ((a), (b)) and at time t ((c), (d)). The images are made from the front side ((a), (c)) and from above ((b), (d)).

angular velocity of this shadow is not uniform. Consequently, the timescale on the horizontal plane below the equatorial sundial (hour lines of a horizontal sundial) has to be determined for each locality of a sundial.

The relation between the angles of the hour lines on the disc and the angles of our lines on the horizontal plane is derived using trigonometry. To derive this relation it is necessary to identify two characteristic planes and three right-angled triangles.

The first characteristic plane is determined by the disc axis and its shadow on the ground at solar noon. This plane is normal to the horizontal plane (figures 9(a) and (b)).

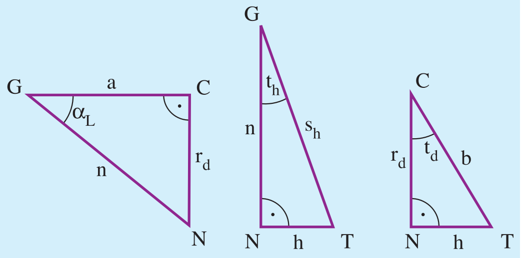

Figure 10. Three right-angled triangles used to derive the relation between angles t_d and t_h.

The second characteristic plane is determined by the disc axis and the direction of its shadow on the ground at the chosen moment of time (figures 9(c) and (d)).

By referring to figure 9, three characteristic right-angled triangles are drawn (see figure 10). It helps to refer to figure 8, as well. The elements of these triangles are as follows.

C—the centre of the disc.

N—the point at the bottom of the disc. This is the point where the noon shadow of the axis on the disc and the noon shadow of the axis on the horizontal plane intersect.

G—the point where the axis touches the horizontal plane.

T—the point where the prolongation of the shadow of the axis on the disc and the shadow of the axis on the horizontal plane intersect at time t.

α_L—the angle between the axis and the horizontal plane. It is equal to the latitude of the dial place.

t_d—the hour angle. This is the mutual angle of the shadow line on the disc at time t and the noon line.

t_h—the face angle. This is the mutual angle of the shadow line on the horizontal plane (face) at time t and the shadow line at the face at noon, which is the south–north line.

Looking at the triangles (figure 10) it is easy to write the following relations:

$$\tan(t_h) = \frac{h}{n} \tag{8}$$

$$\sin(\alpha_L) = \frac{r_d}{n} \tag{9}$$

$$\tan(t_d) = \frac{h}{r_d}. \tag{10}$$

Table 1. The values of hour angles and face angles for $\alpha_L = 40°$.

Sundial time	t_d	t_h
12 Noon	0°	0°
11:00 am and 1:00 pm	15°	9.77°
10:00 am and 2:00 pm	30°	20.36°
9:00 am and 3:00 pm	45°	32.73°
8:00 am and 4:00 pm	60°	48.07°
7:00 am and 5:00 pm	75°	67.37°
6:00 am and 6:00 pm	90°	90.00°

By multiplying relations (9) and (10), and comparing with (8), we obtain

$$\sin(\alpha_L) * \tan(t_d) = \frac{h}{n} = \tan(t_h). \tag{11}$$

Therefore

$$t_h = \arctan(\sin(\alpha_L) * \tan(t_d)). \tag{12}$$

By choosing the latitude α_L we easily evaluate from relation (12) the values of the angles t_h for six values of the angle $t_d = z \times 15°$, where $z = 1, 2, \ldots 6$. The values corresponding to $\alpha_L = 40°$ are given in table 1.

Conclusions

Globes used in school or university education, either in closed or open space, can be used to simulate the Earth's orientation, like Rishpon's globe in the Clore Garden of Science at Rehovot in Israel. We hope to have shown in this article that such globes could be a very useful and valuable teaching tool in earth science, physics, geometry, astronomy and history of science courses. With an extended axis and appropriate time scale on the ground, such a globe would serve as a horizontal sundial. With pins around the equator it would become a spherical sundial.

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